Nuclear Physics from Lattice QCD

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Collaborators

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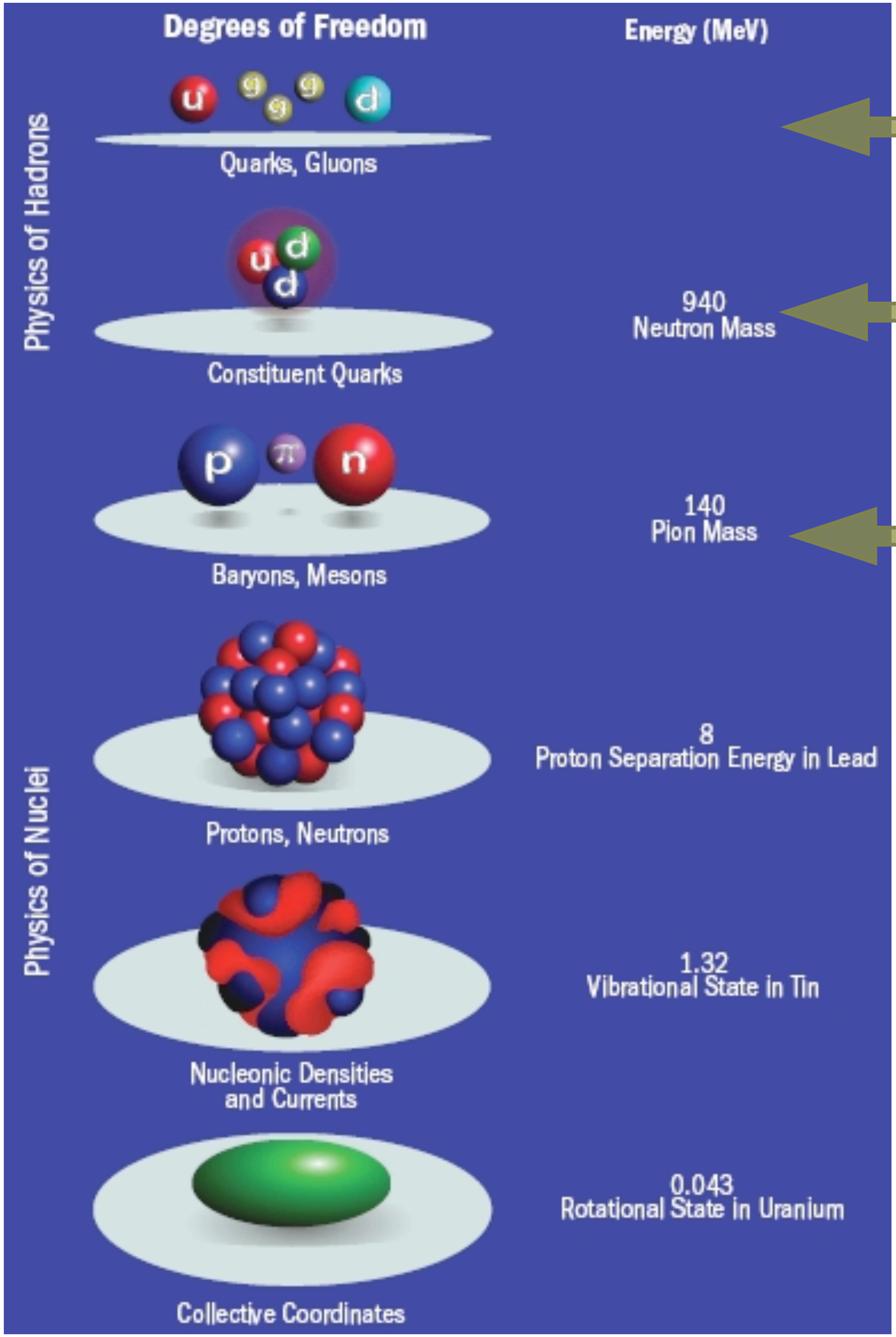
Motivation-Overview

- Study Multi-hadron systems
- Applications:
  - Nuclear physics (A=**2**, 3, 4 ...)
  - Spectroscopy (excited states, multi-quark hadrons etc.)
- Methodology: Use finite volume
  - Extract scattering phase shifts
  - Extract multi-hadron interaction properties
- Results and Conclusions

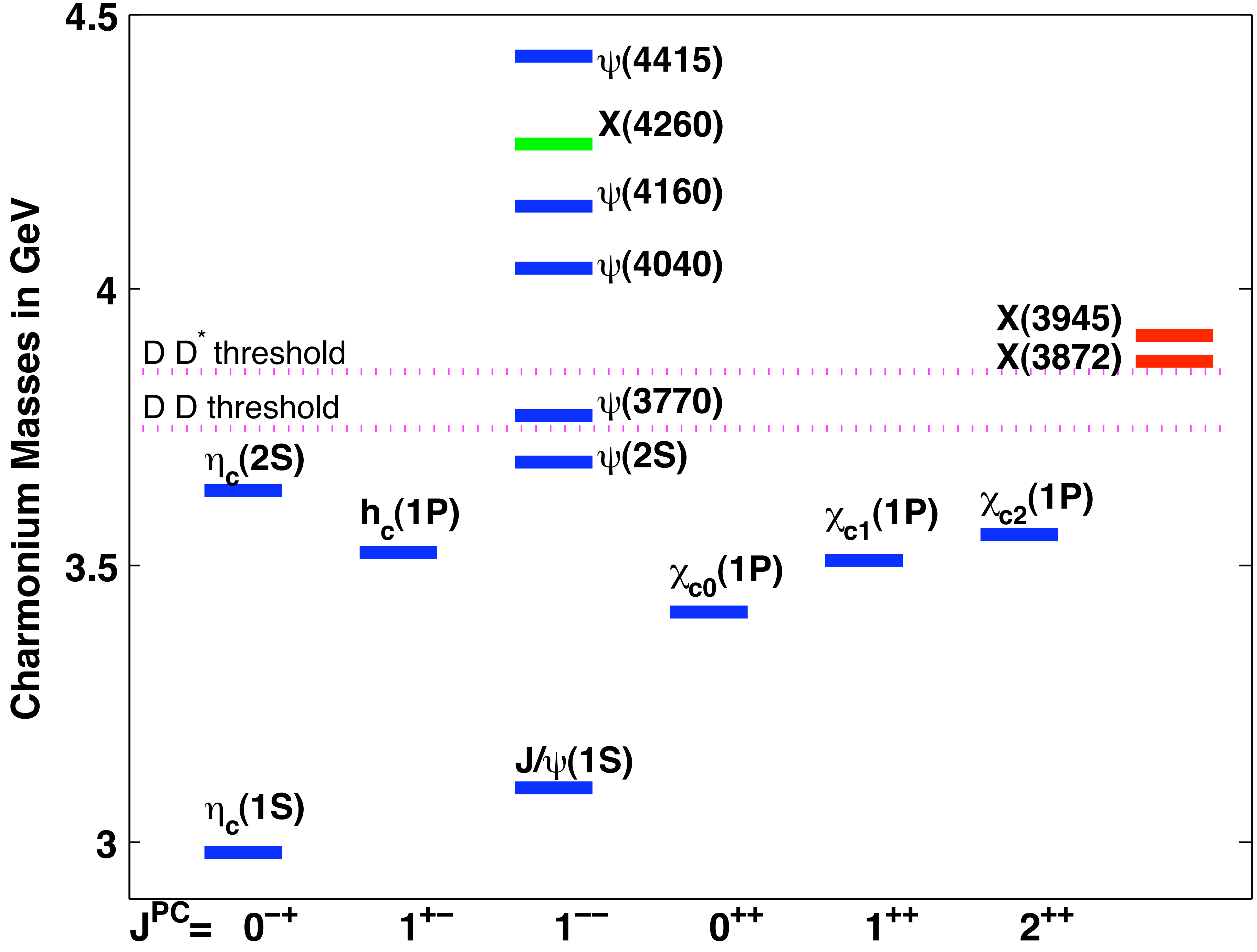


QCD

Hadron Structure

Hadron
Interactions

Charmonium Spectrum



PDG 2009

Hadron Scattering

- Scattering processes from Lattice QCD are not straight forward
- Miani-Testa no-go theorem ('90) [and C. Michael '89]
- Infinite Volume:

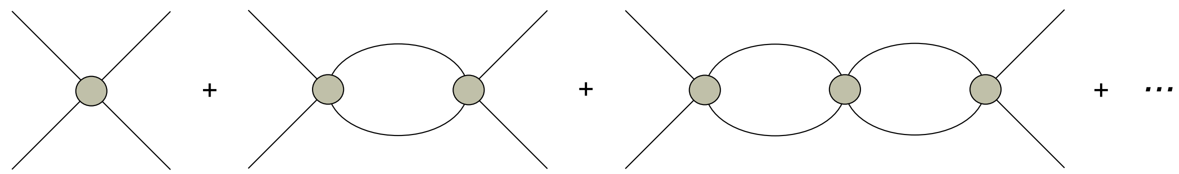
  Euclidean $\not\rightarrow$ Minkowski

- Finite volume: discrete spectrum
  - Avoids Miani-Testa no-go theorem [M. Luscher]

Scattering on the Lattice

Luscher

Scattering amplitude:

$A(p) =$ 

$$A(p) = \frac{4\pi}{m} \frac{1}{p\,cot\delta \; - \; i\,p}$$

At finite volume one can show:

$$\Delta E_n \;\equiv\; E_n \;-\; 2m \;=\; 2\sqrt{p_n^2 \;+\; m^2} \;-\; 2m$$

p_n solutions of:

$$p\cot\delta(p) \;=\; \frac{1}{\pi L}\,\mathbf{S}\left(\frac{p^2L^2}{4\pi^2}\right)$$

$$\mathbf{S}\,(\,\eta\,) \;\equiv\; \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta} \;-\; 4\pi\Lambda$$

Effective range expansion:

$$p\cot\delta(p) = \frac{1}{a} + \frac{1}{2}rp^2 + \ldots.$$

a is the scattering length

Luscher Formula

Energy level shift in finite volume:

$$\Delta E_n \equiv E_n - 2m = 2\sqrt{p_n^2 + m^2} - 2m$$

p_n solutions of:

$$p \cot \delta(p) = \frac{1}{\pi L} \mathbf{S}\left(\frac{p^2 L^2}{4\pi^2}\right)$$

$$\mathbf{S}(\eta) \equiv \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2 - \eta} - 4\pi\Lambda$$

$$p_n \cot \delta(p_n) = \frac{1}{a} + \cdots$$

$$\frac{1}{a} = \frac{1}{\pi L} S\left(\frac{p_0^2 L^2}{4\pi^2}\right) + \cdots$$

Expansion at p~0 :

$$\Delta E_0 = -\frac{4\pi a}{m L^3}\left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L}\right)^2\right] + \mathcal{O}\left(\frac{1}{L^6}\right)$$

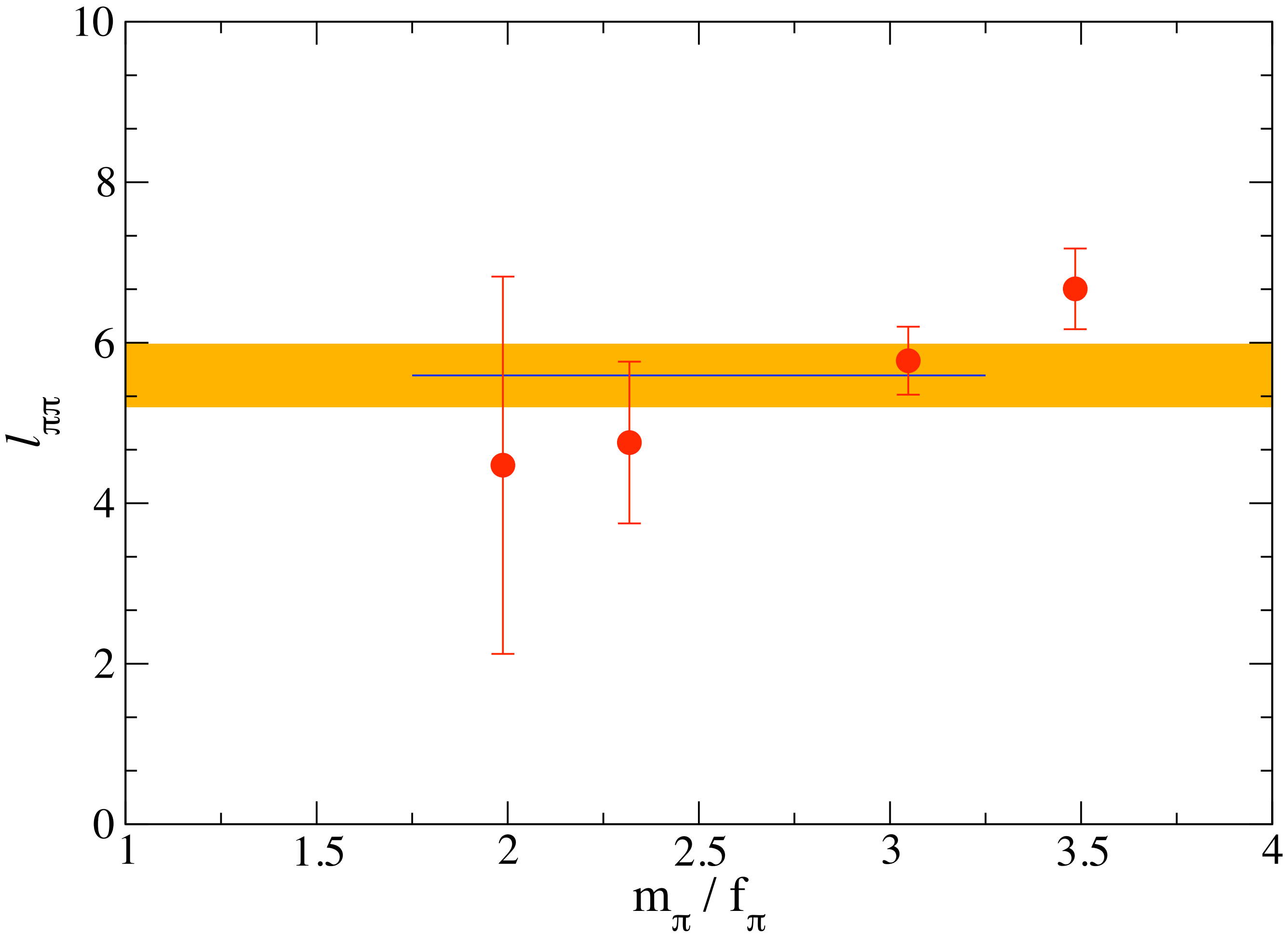
a is the scattering length

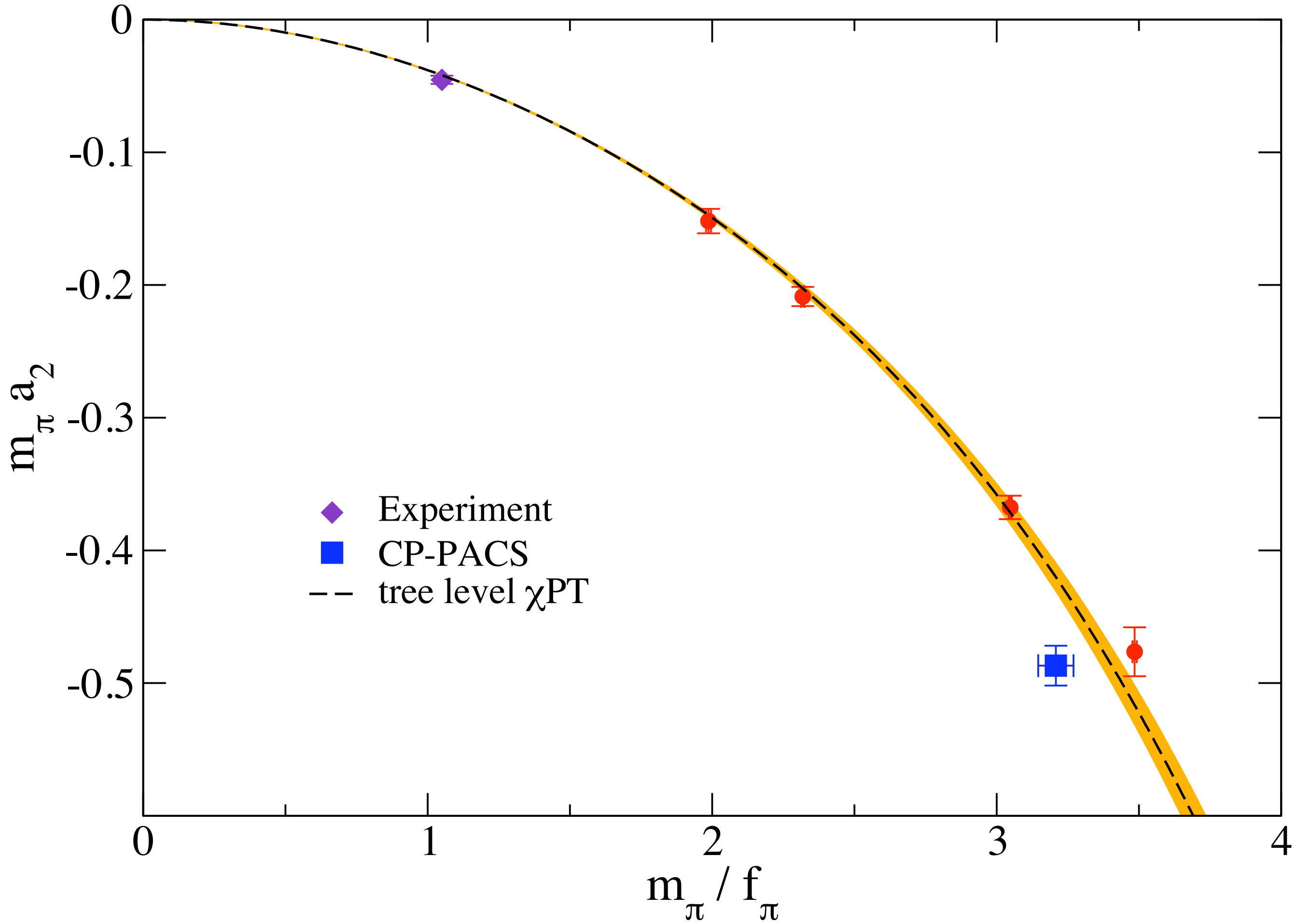
c_1 and c_2 are universal constants

Technicalities

- How well can we compute energy levels in LQCD?
  - Ground state:
    - Relatively easy for mesons
    - Baryons are hard and get harder as their number grows
  - Excited states are more demanding but methods do exist
- Theoretical background
  - Two body problems are well understood [Lucher]
  - Multi-mesons recently done [Beane et.al. PRD76;074507, 2007, Detmold et.al. PRD77:057502,2008]
  - Coupled channels: Works begin to appear [Lage et.al. **arXiv:0905.0069**]
  - More than two baryons: Work in progress.... [Luu **arXiv:0810.2331v1**]
  - Still a lot needs to be done!

I=2 Pion Scattering



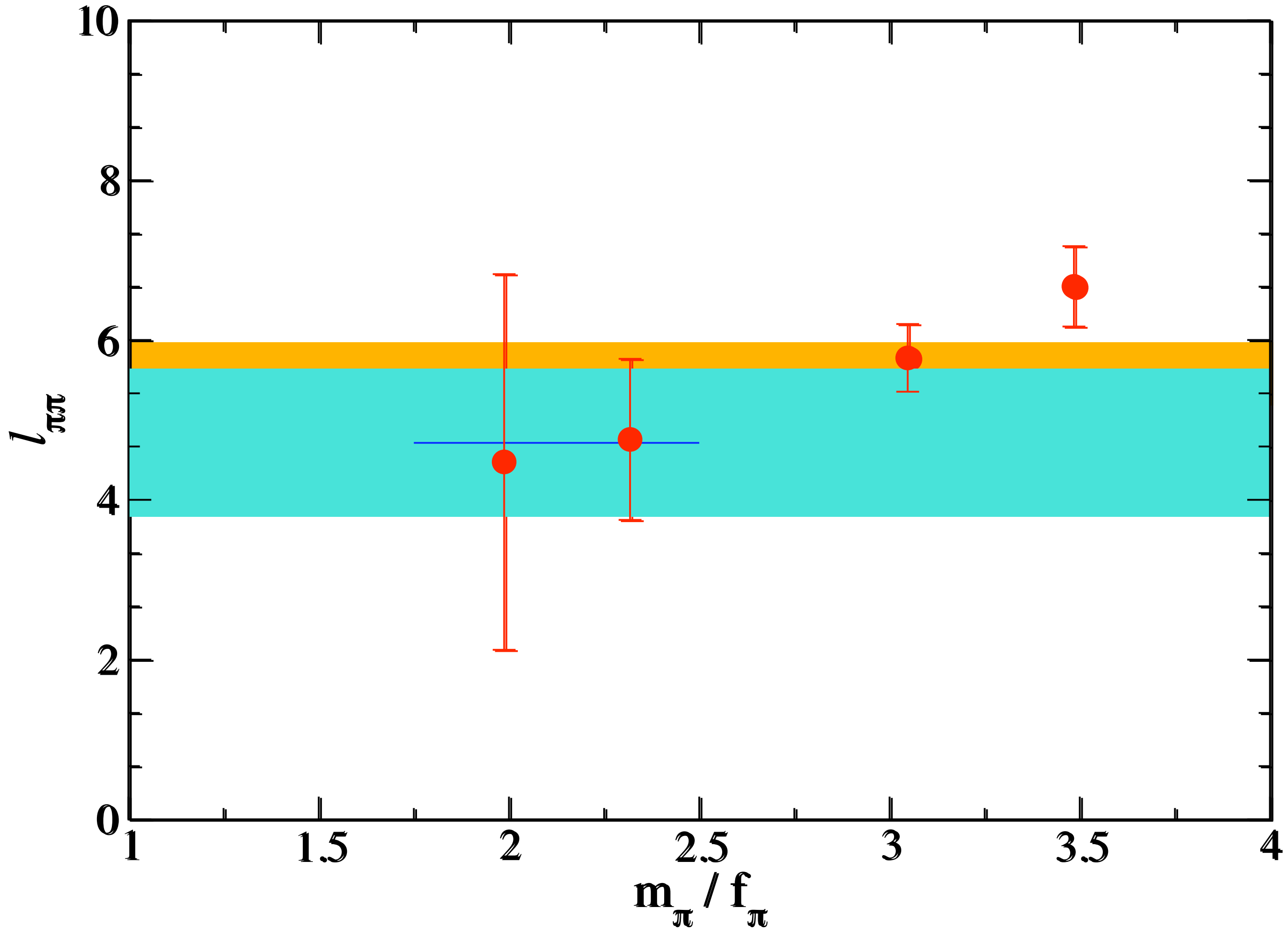


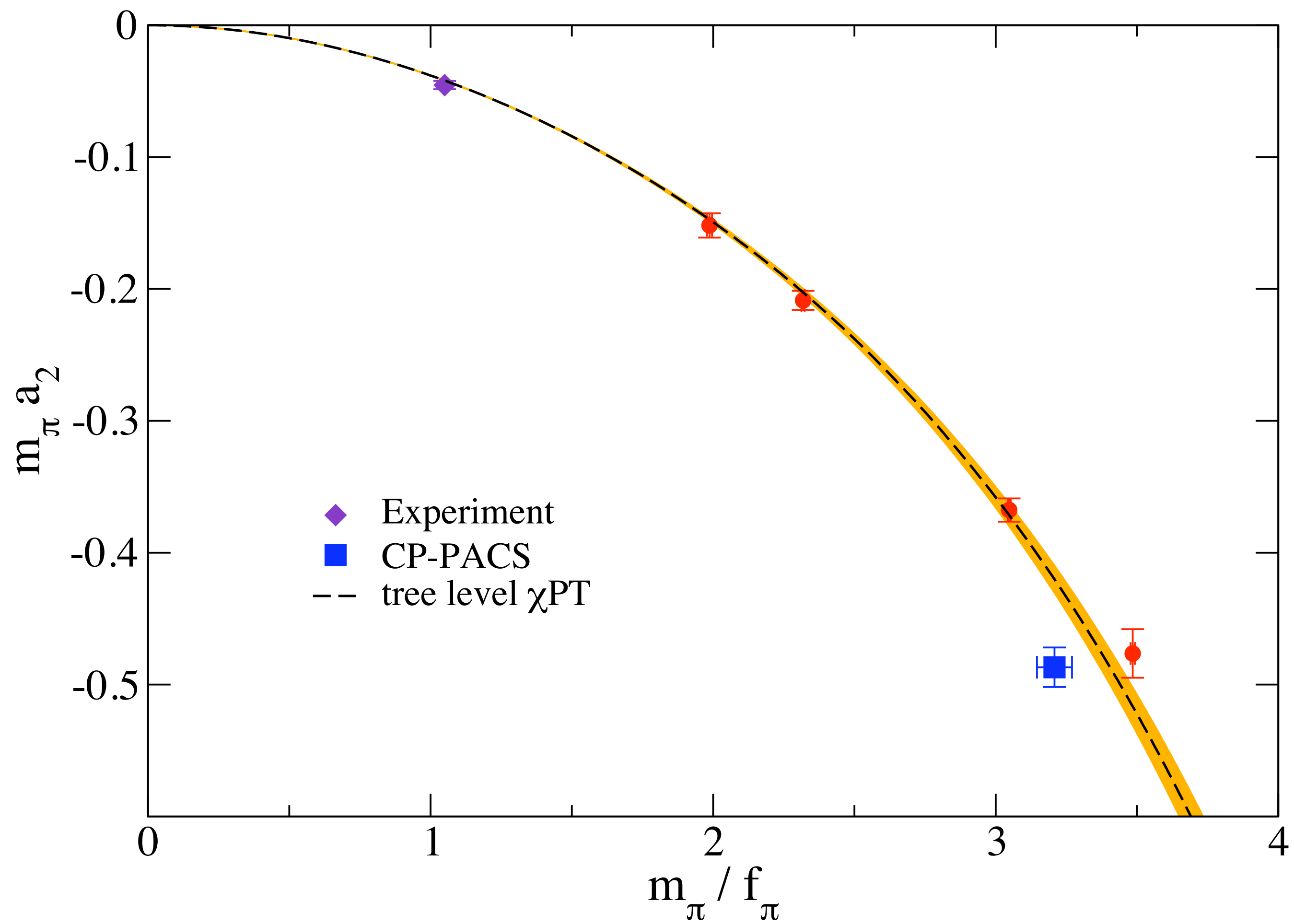
SU(2) ChiPT

$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left[1 + \frac{m_\pi^2}{16\pi^2 f_\pi^2}\left[3\log\left(\frac{m_\pi^2}{f_\pi^2}\right) - 1 - l_{\pi\pi}(f_\pi)\right]\right]$$

- Physical point -- Lighter three point fit: $m_\pi a_2 = -0.04196(12)$
- Physical point -- Lighter two point fit: $m_\pi a_2 = -0.04223(28)$
- Physical point -- Quadratic fit (higher order): $m_\pi a_2 = -0.0426(4)$

I=2 Pion Scattering





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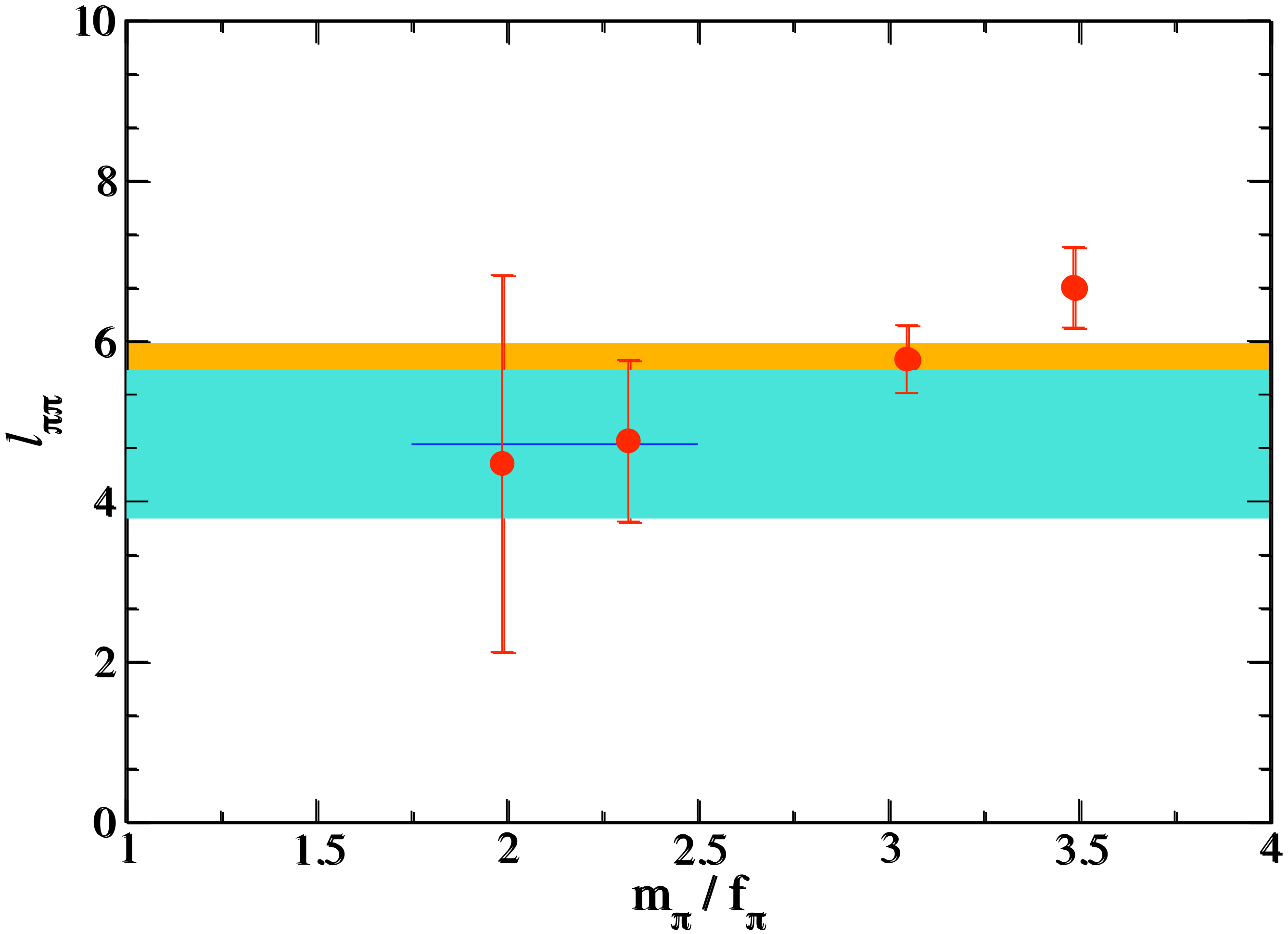

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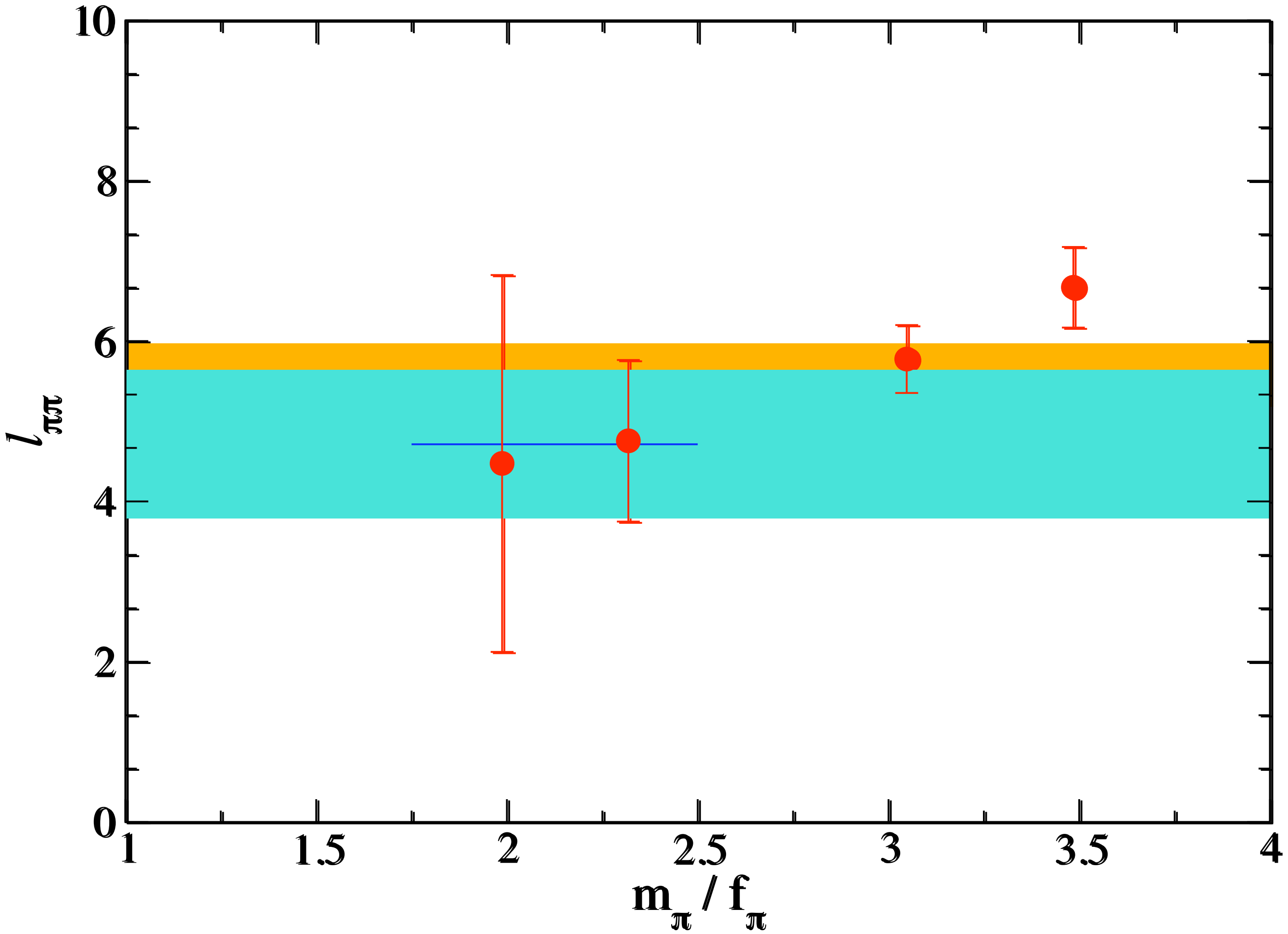

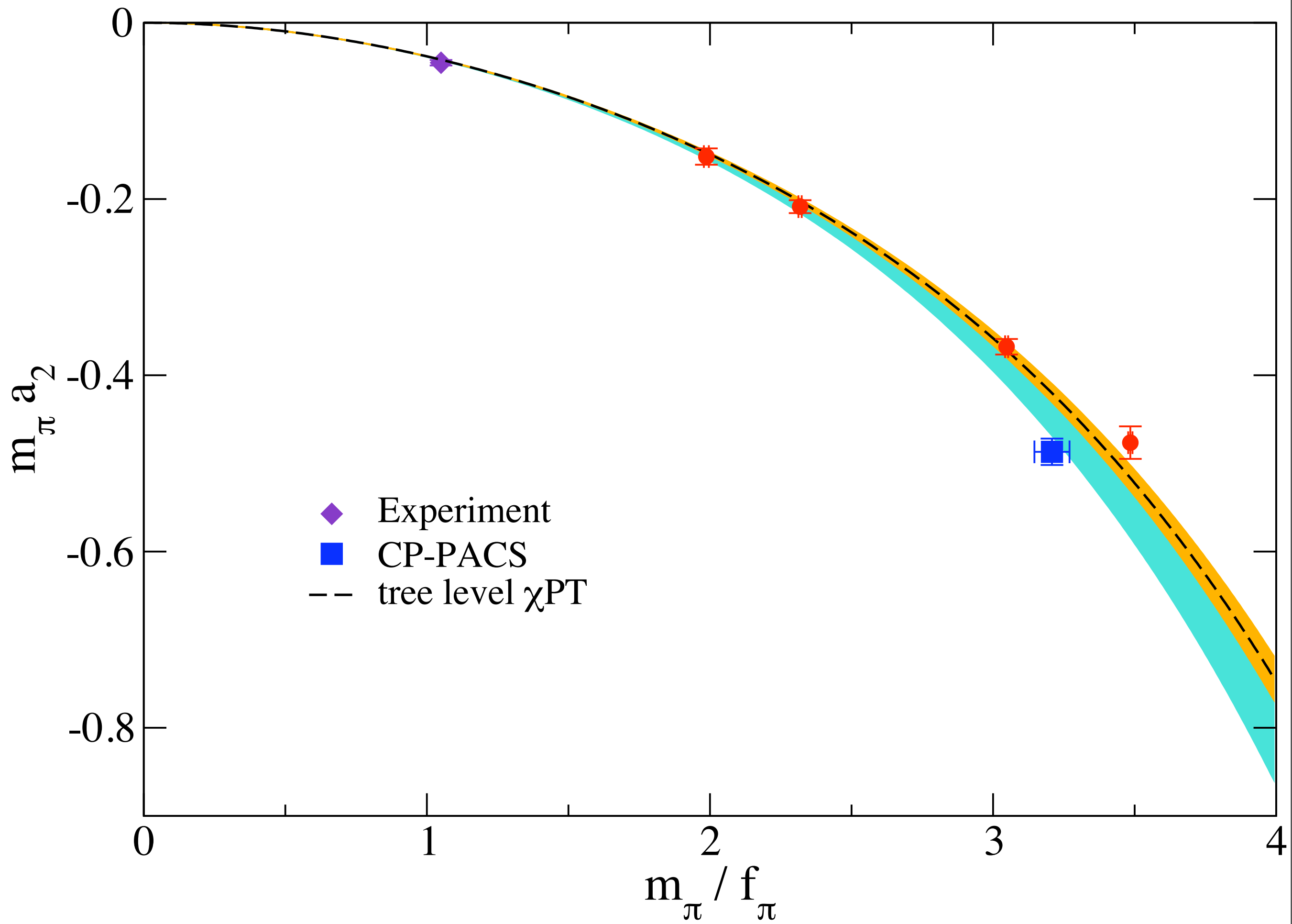

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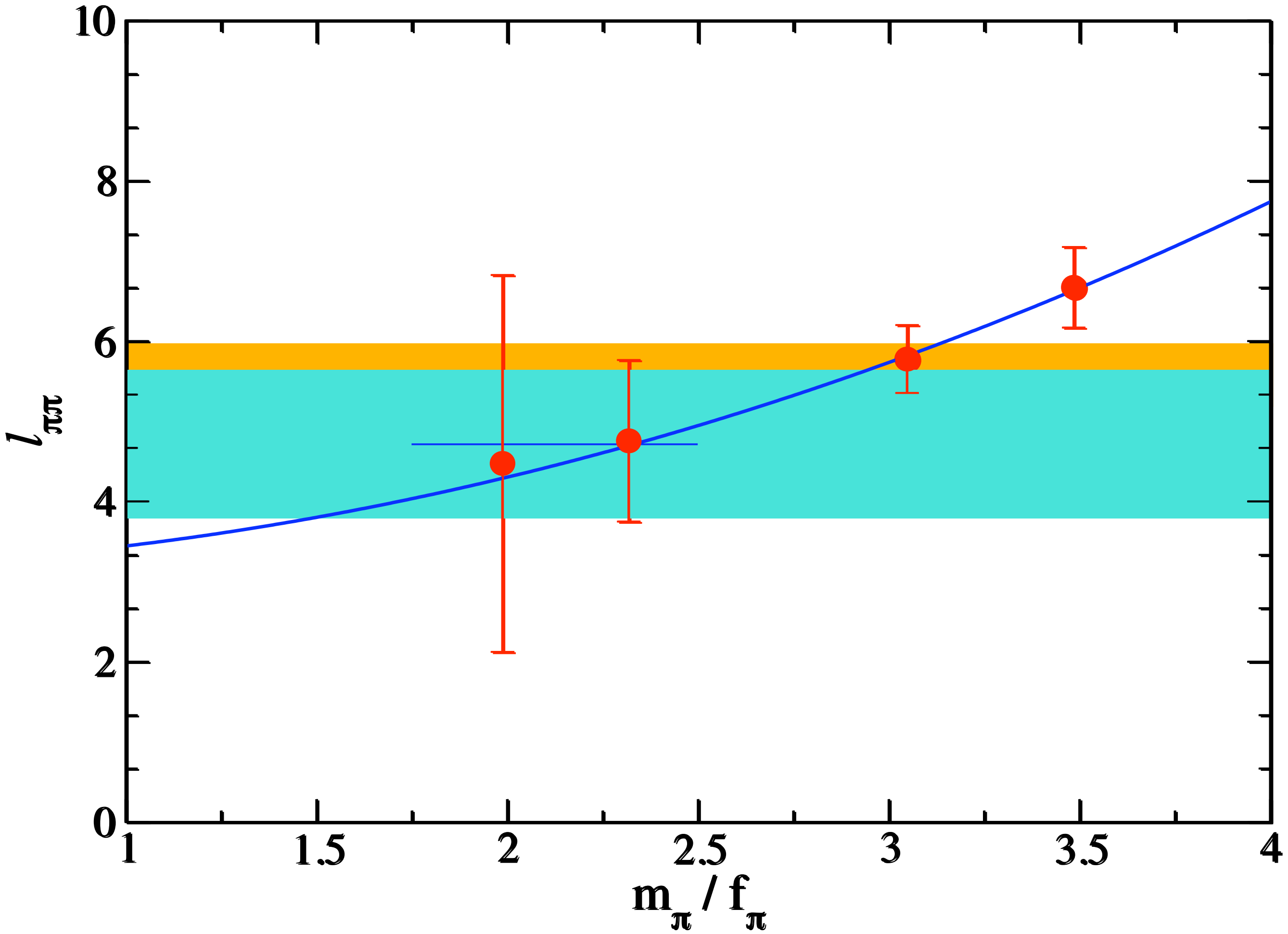

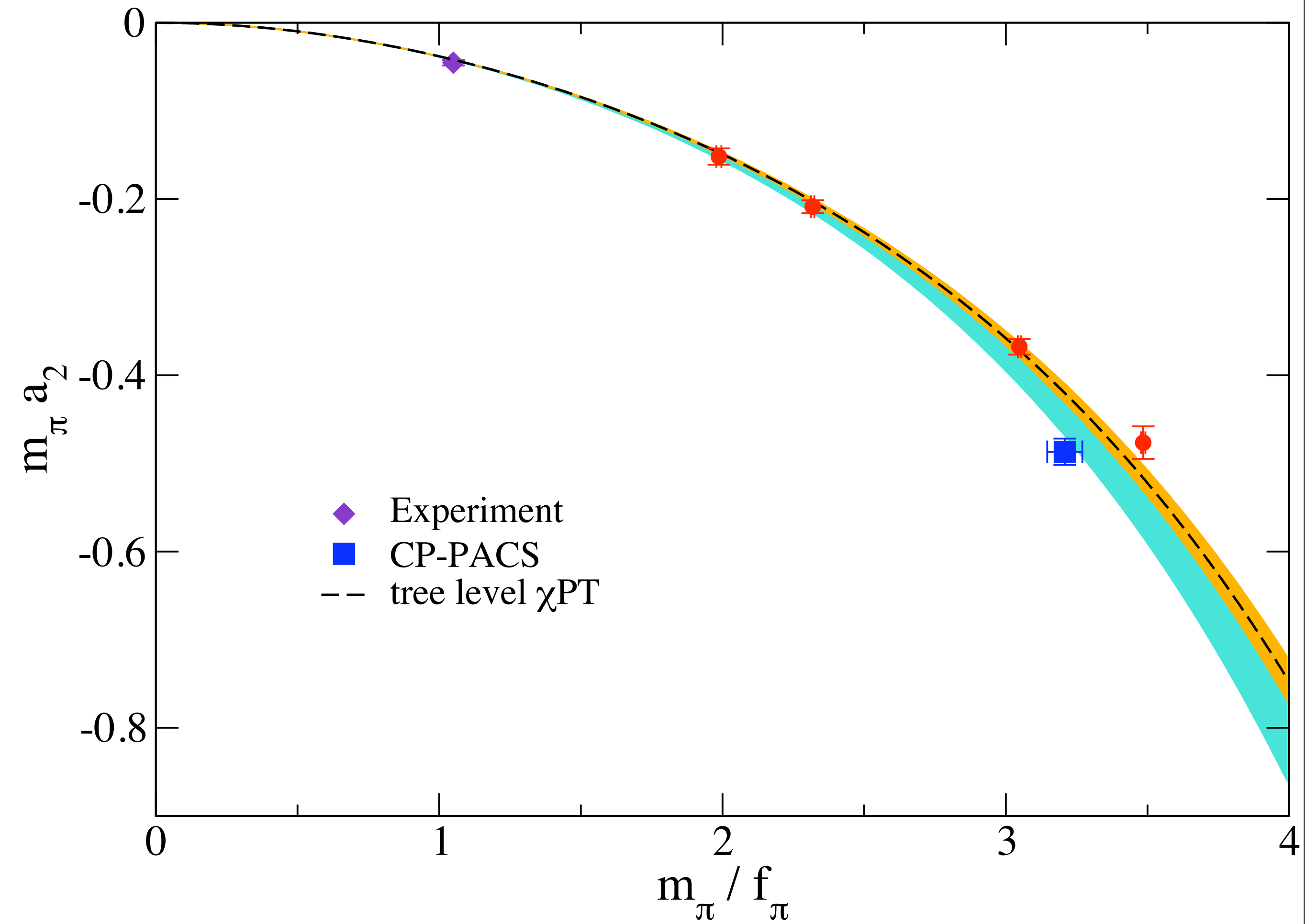

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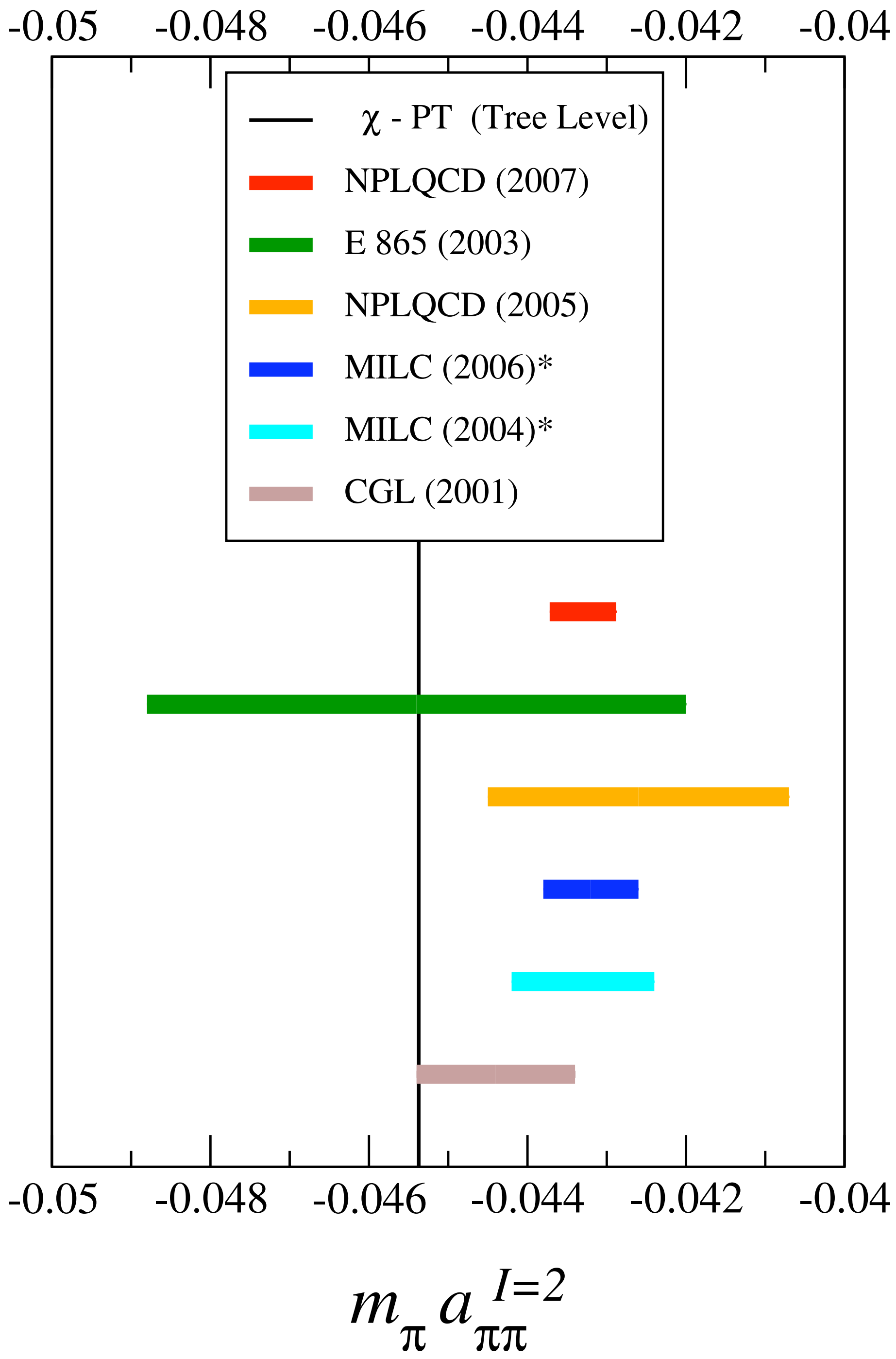
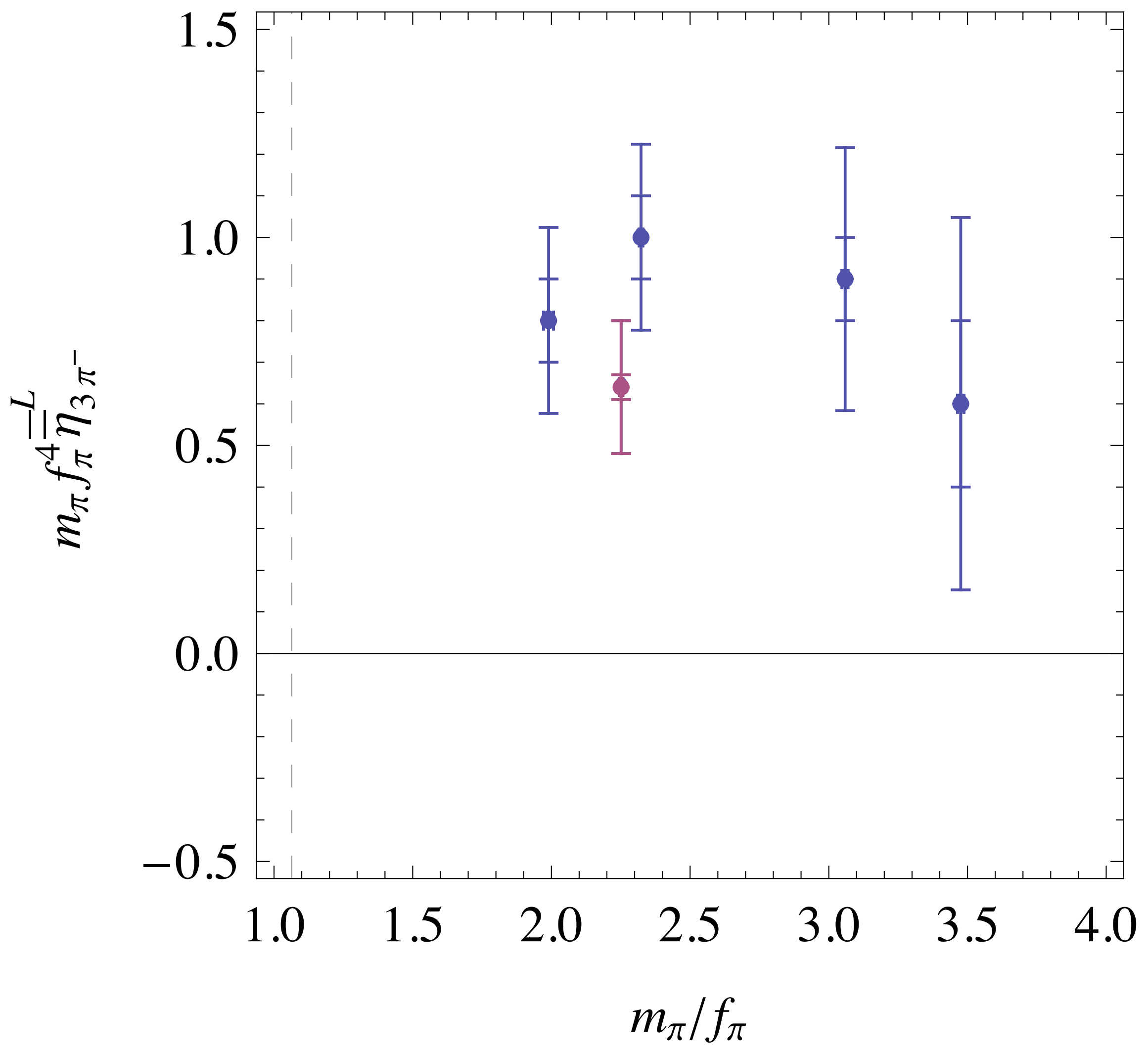
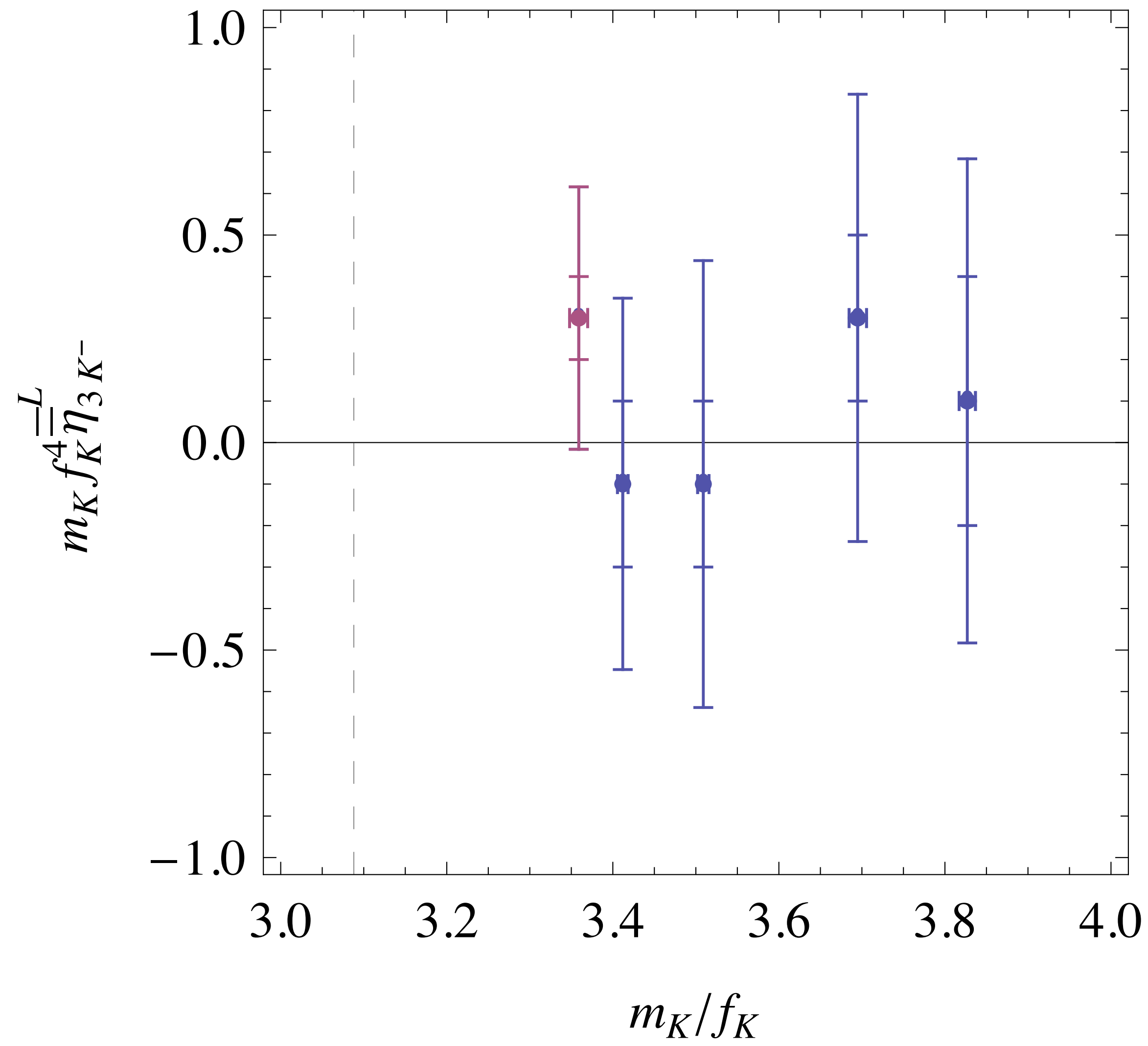
World results on I=2 π-π scattering lengths



Three meson interaction

NPLQCD: **Phys.Rev.D78 (054514,014507) 2008**

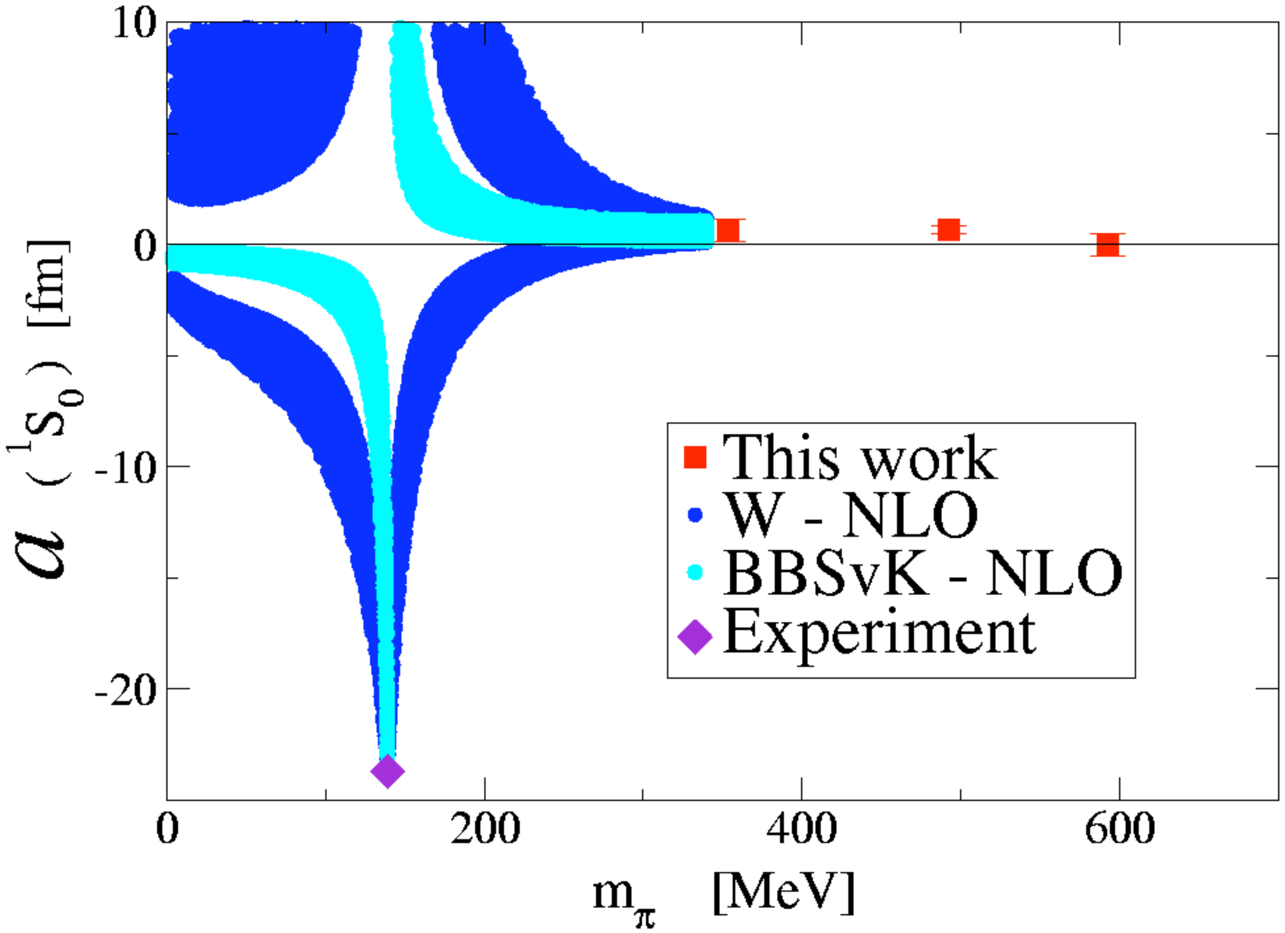


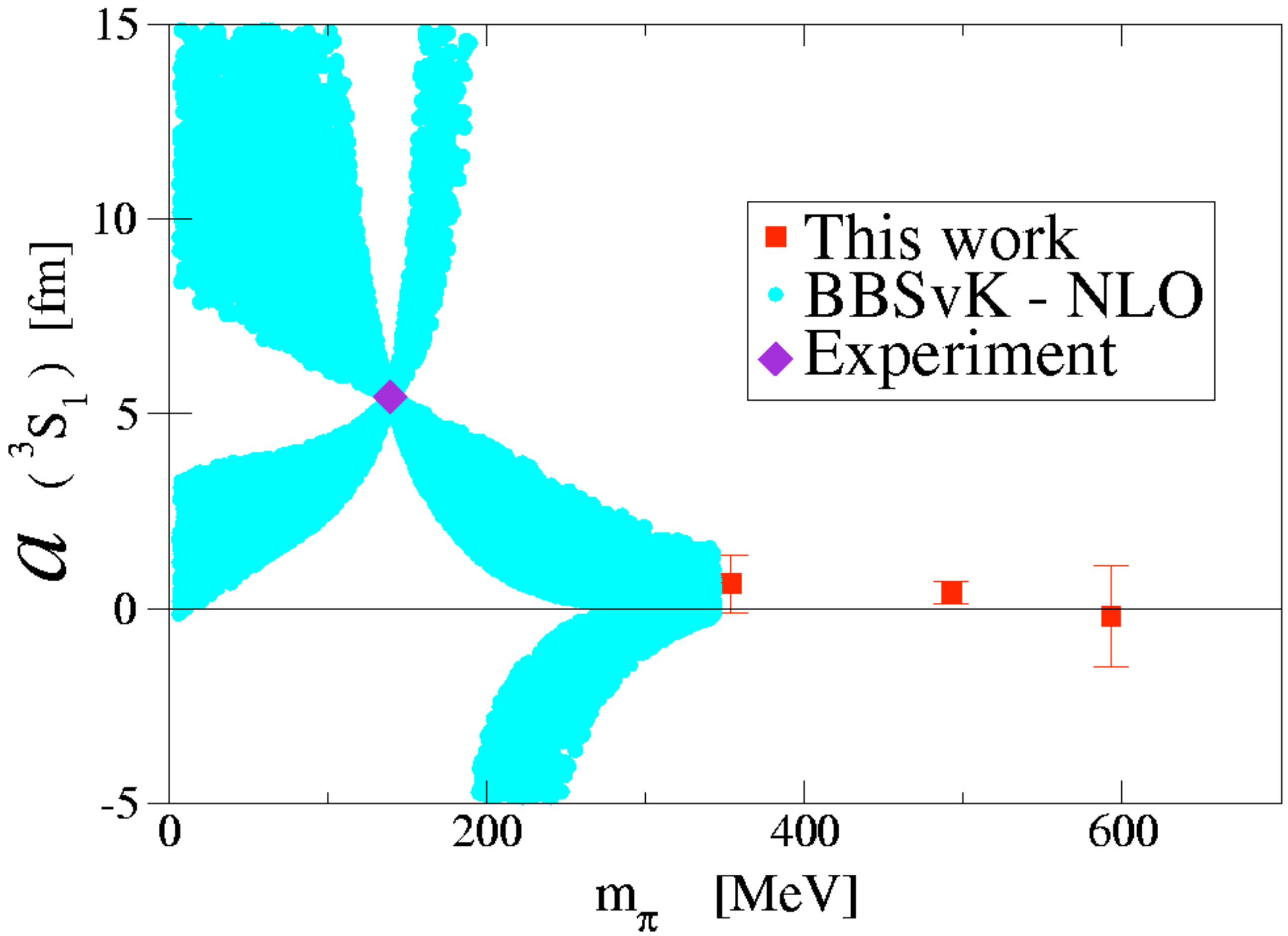


- Three pion interaction is non-zero
- Three kaon interaction vanishes

Nucleon-Nucleon

NPLQCD: **Phys.Rev.Lett.97 2006**





BBSvK: Beane Bedaque Savage van Kolck '02

W: Weinberg '90;Weingberg '91; Ordonez et.al '95

Fukugita et al. '95: Quenched heavy pions

Signal to Noise ratio for correlation functions (mesons)

$$C(t) = \langle m(t)\bar{m}(0)\rangle \sim Ae^{-M_m t}$$

$$var(C(t)) = \langle m\bar{m}(t)m\bar{m}(0)\rangle \sim Ae^{-2M_m t} + Be^{-2M_\pi t}$$

$$StoN = \frac{C(t)}{\sqrt{var(C(t))}} \sim Ae^{-(M_m - M_\pi)t}$$

- For pseudo-scalar mesons the signal does not deteriorate at large time

Signal to Noise ratio for correlation functions (baryons)

$$C(t) = \langle N(t)\bar{N}(0)\rangle \sim Ee^{-M_N t}$$

$$var(C(t)) = \langle N\bar{N}(t)N\bar{N}(0)\rangle \sim Ae^{-2M_N t} + Be^{-3m_\pi t}$$

$$StoN = \frac{C(t)}{\sqrt{var(C(t))}} =\sim Ae^{-(M_N - 3/2 m_\pi)t}$$

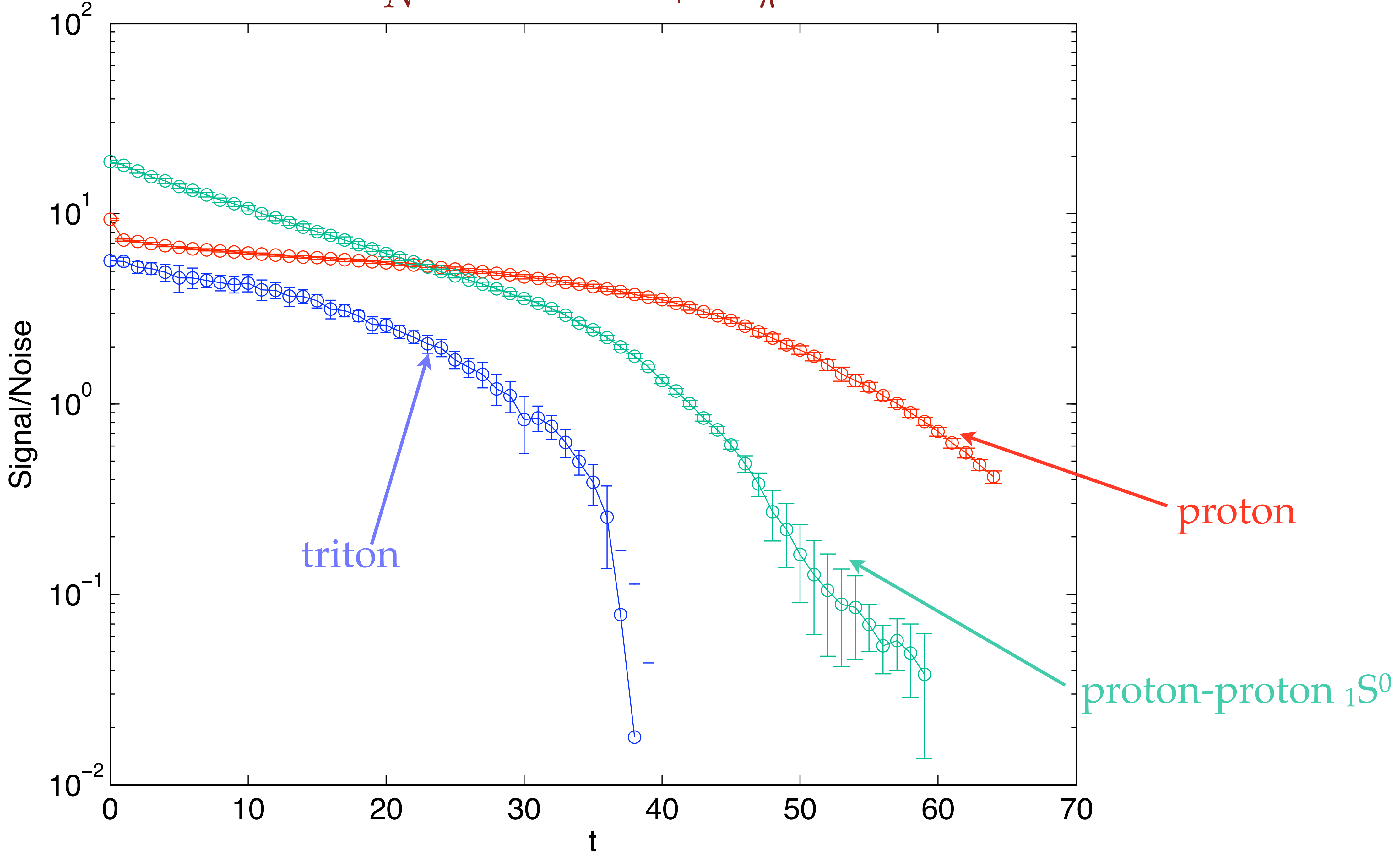
- The signal to noise ratio drops exponentially with time
- The signal to noise ratio drops exponentially with decreasing pion mass
- For two baryons: StoN(2N) = StoN(1N)^2

High Statistics runs

- Improved Wilson fermions on anisotropic lattices
- Single pion mass: 390MeV
- Very high statistics (~300x10^3 correlators)
- Goals
  - Get clean signals
  - Investigate methods of extracting masses from correlation functions
  - Check feasibility of 3 body calculations

Signal to Noise Ratio

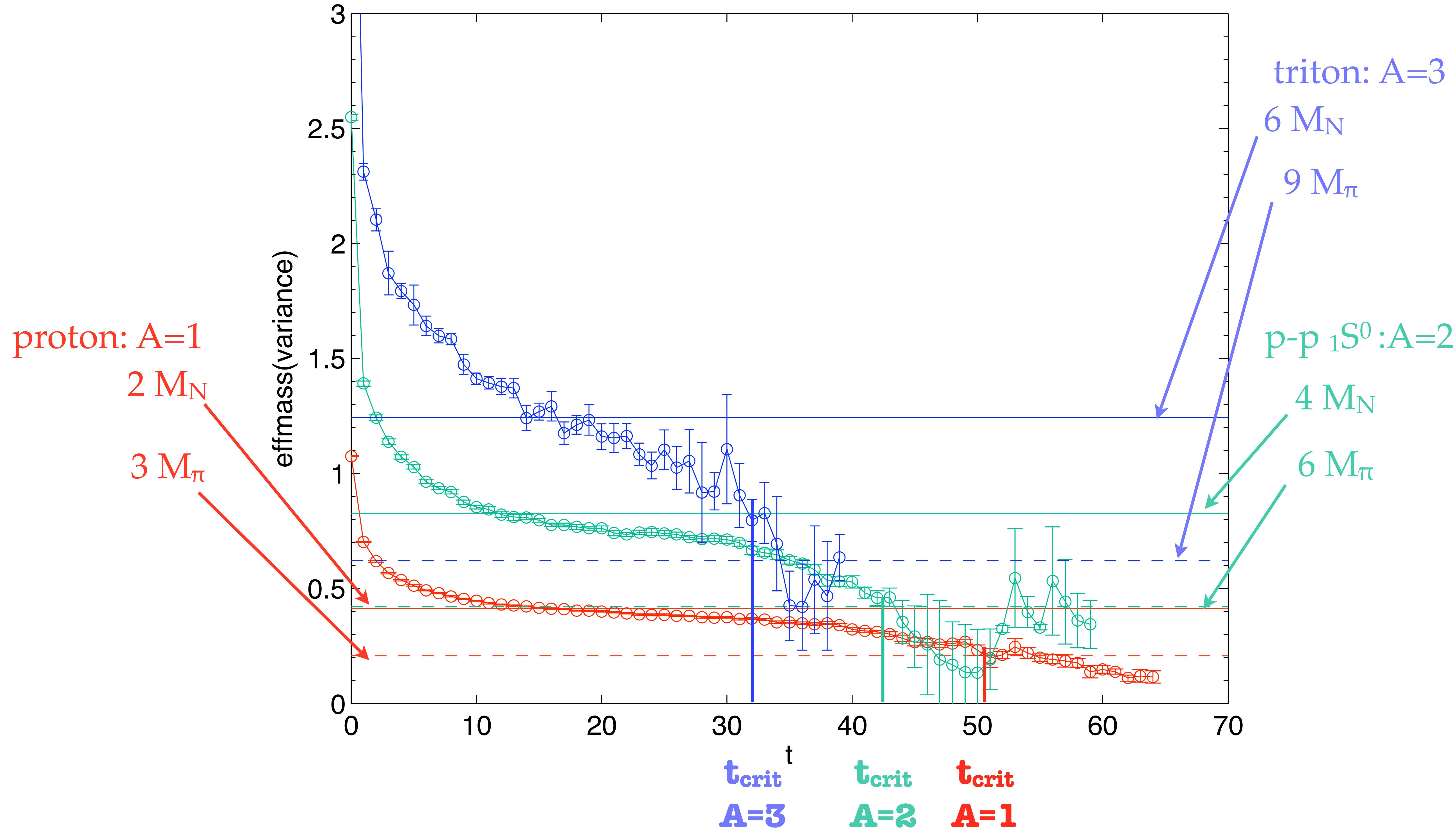
$$var \sim C_N e^{-2AM_N t} + C_\pi e^{-3AM_\pi t}$$



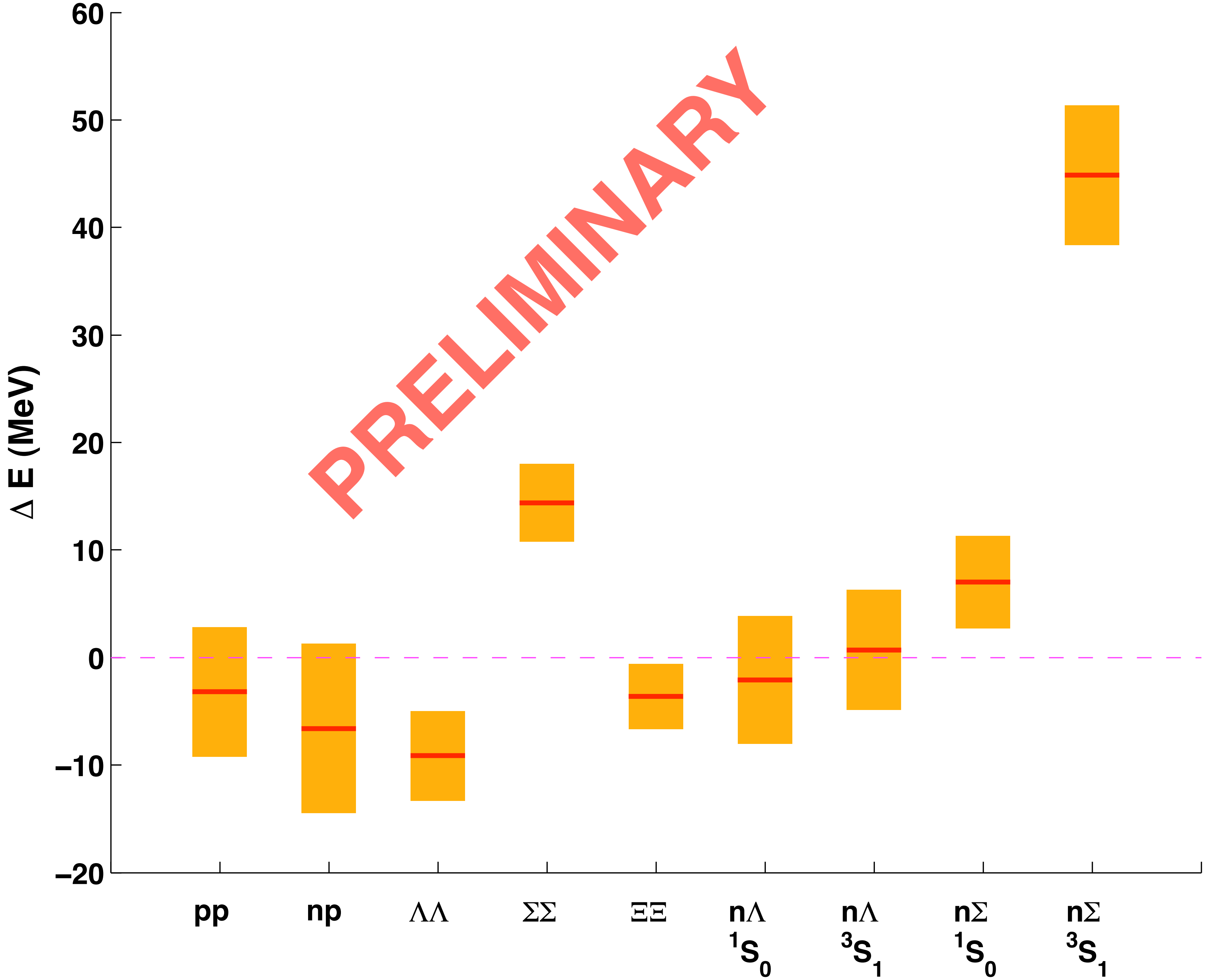
- Exponential loss of signal at large time
- Slopes become larger as baryon number increases

Exponential Slope of the variance

$$var \sim C_N e^{-2AM_N t} + C_\pi e^{-3AM_\pi t}$$

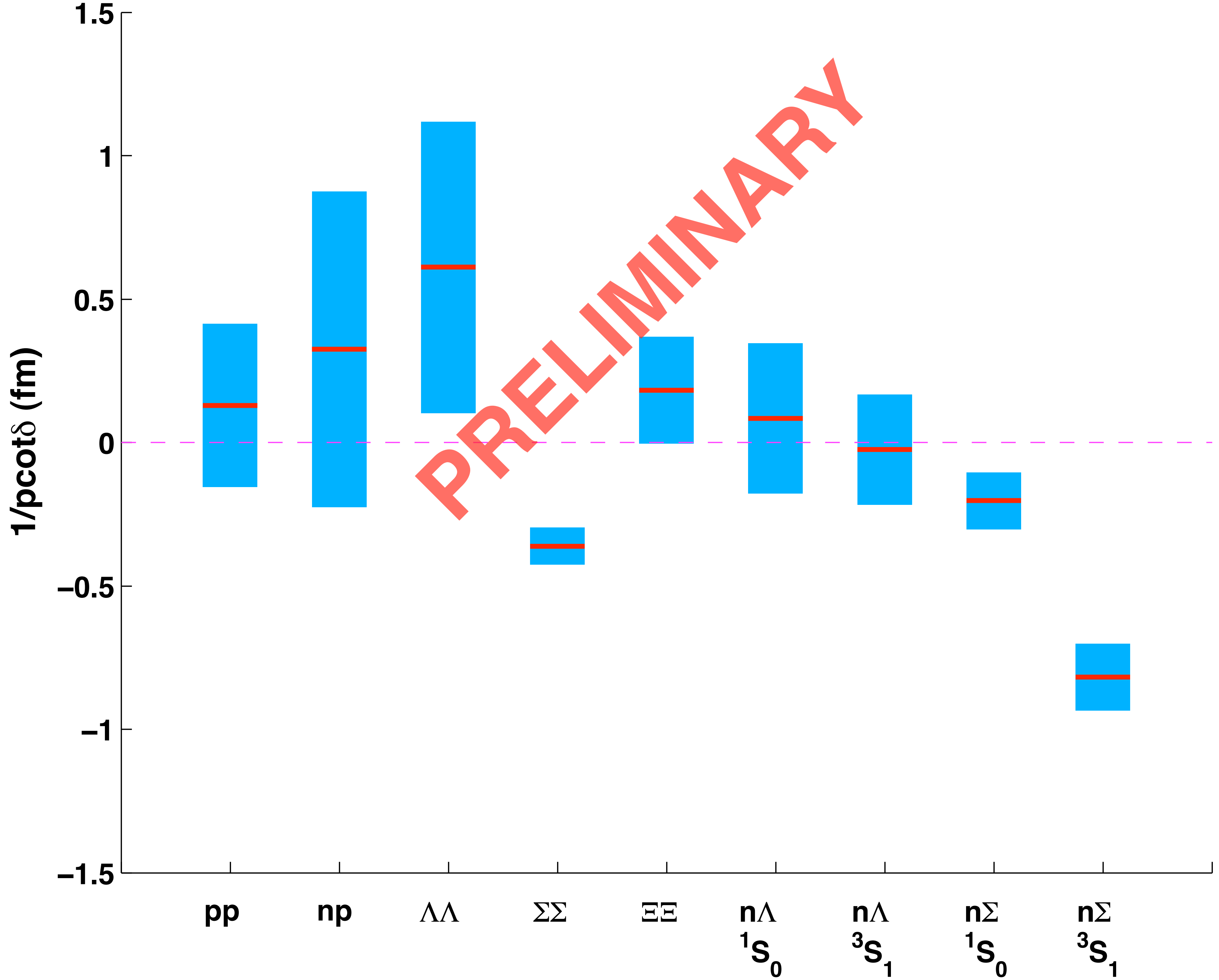


Energy Shifts



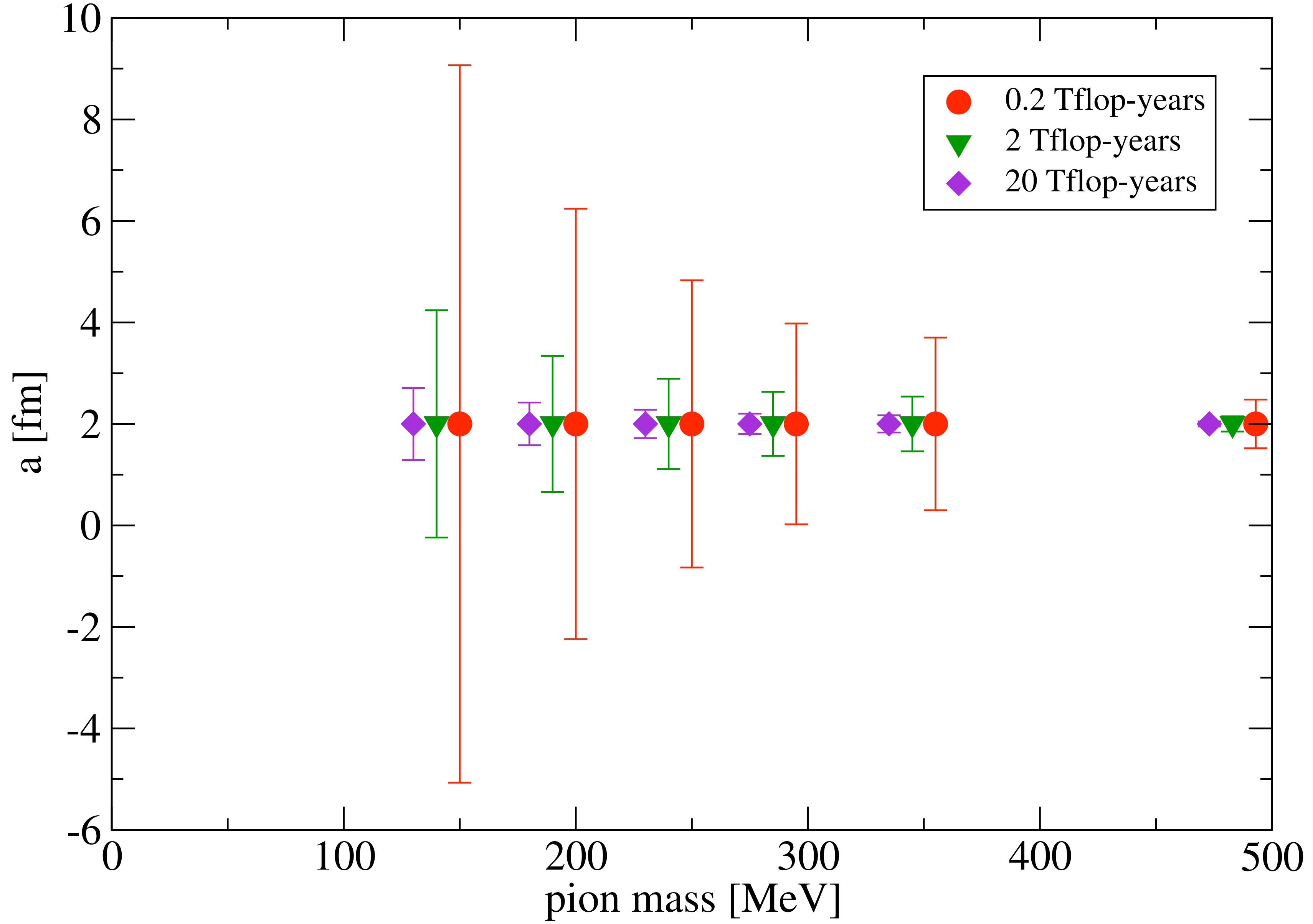
pion mass: 390MeV

Scattering Lengths



pion mass: 390MeV

Nucleon-Nucleon Interactions: Projected errors



- Errors on scattering nucleon-nucleon scattering length as function of computational resources
- Only cost for correlation function calculation presented
- Includes expected speedup from eigCG [Stathopoulos, KO 2007]

Summary

- Mesonic (pseudo-scalar) many body systems are successfully studied in LQCD
  - Scattering lengths
  - Three body interaction
- Baryons are hard
  - Statistical noise require high statistics calculations
- Advent of peta-scale computing together with new algorithmic developments will provide interesting results over the next few years